Localization of eigenstates in a modified Tomonaga-Luttinger model

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We study the localization in the Hilbert space of a modified Tomonaga-Luttinger model. For the standard version of this model, the states are found to be extended in the basis of Slater determinants, representing the eigenstates of the non-interacting system. The linear dispersion which leads to the fact that these eigenstates are extended in the modified model is replaced by one with random level spacings modeling the complicated one-particle spectra of realistic models. The localization properties of the eigenstates are studied. The interactions are simplified and an effective one-dimensional Lloyd model is obtained. The effects of many-body energy correlations are studied numerically. The eigenstates of the system are found to be localized in Fock space for any strength of the interactions, but the localization is not exponential.

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I. INTRODUCTION

The concept of Anderson localization in the Fock space of many-body systems has been recently adopted for the studies of interacting electrons in finite-size conductors. The traditional field of localization consists of the investigation of electronic motion in disordered solids. These studies were initiated by Anderson [1] who considered tight-binding Hamiltonians on real-space lattices with random on-site energies. The behavior of the system was found to depend on the parameter $Z = W/V$, where W is the typical variance of the random potential and V is the hopping matrix element. For Z above some critical value the eigenfunctions of the Hamiltonian are exponentially localized with a characteristic scale ξ, called the localization length, which leads to the absence of diffusion of electrons. The Anderson problem inspired numerous studies (for reviews see [2–4]). The scaling theory of localization [5] predicted the strong dependence of the localization properties on the dimension of the lattice. It was found, for instance, that in one spatial dimension the states are always localized for any strength of the hopping.

The ideas of localization can be extended to the investigation of the properties of interacting many-body Hamiltonians in the basis of eigenstates of the corresponding free system. The central problem considered there is the mixing of a particular state representing one particle excited above the Fermi surface with many-particle states responsible for emergence of a quasi-particle described by the usual Landau Fermi liquid theory. For this purpose the analogy between the one-particle and the many-body problems was demonstrated in [6] by mapping the Hilbert space of interacting electrons onto an effective tight-binding model. Each site of the effective lattice represents the eigenstate of the non-interacting system and the bonds represent the interaction matrix elements. After several approximations, which consist mainly in neglecting the possibility of closed loops on this effective lattice, the resulting model was identified with the Cayley tree with branching number K depending on the energy ϵ of the initial excitation, representing a particle excited above the Fermi sea. The Cayley tree model was previously studied [7] and was found to undergo a localization transition for the value of parameter $Z = Z_c \approx K \ln K$. In the context of the original many-electron problem, the system is in the localized regime of the many-particle states on the states of the non-interacting system for energy below the lower threshold $\epsilon^{**} = \Delta\sqrt{g/\ln g}$, where Δ is the one-particle level-spacing and g is dimensionless conductance, i.e. conductance in units of $e^2/\hbar$. It was also argued that until the excitation energy ϵ reaches another critical value $\epsilon^* = \Delta\sqrt{g} > \epsilon^{**}$ the states are not completely delocalized, i.e. they do not mix large number of non-interacting states, but rather a small portion of them, corresponding to a subtree of the whole lattice. The last result was not confirmed in the subsequent studies [8] of this model by super-symmetry methods. It was found there that the states are strongly correlated above the threshold $\epsilon > \epsilon^{**}$.

In more recent numerical studies of the interacting models [9,10] it was observed that the choice of the type of the lattice affects the localization-delocalization transition point. The crossover between the spectral statistics and the properties of eigenstates such as inverse participation ratio were shown to be sensitive to the type of the lattice. It was doubted that the actual transition and the properties of states in the delocalized phase can be accounted for properly using the pure tree-like structure of the lattice. It was claimed in [11] that the infinite tree-like lattice used in [6] may be inadequate for description of the splitting of the quasi-particle peak into many-body constituents. The finiteness of the lattice used in this work results in the absence of the sharp transition from localized to delocalized states. In the numerical work [12], where the possibility of closed loops on the lattice was taken into account using the random-matrix theory no delocalization transition was observed. Rather the participation ratio and spectral width of the eigenstates of the interacting system exhibited a smooth crossover from almost localized to delocalized states.

The exact solution for the delocalized phase (which corresponds to the strongly interacting regime) is not available in these studies in any limit . It would be useful to have a model which can clarify the nature of the extended states in the strongly interacting regime. The candidates for such a system are of course one-dimensional interacting system whose analytical solution is available for any strength of the interactions. The present work was inspired by the observation that the eigenstates of the simplest interacting problem, namely the chiral Tomonaga-Luttinger model [13] are extended in the effective lattice of the non-interacting Slater determinants. This model is usually solved by the bosonization technique, which can be viewed as a generalization of the solutions by Fourier transform of tight-binding models without disorder. This analogy can be traced more explicitly if one considers the Hilbert space spanned by the Slater determinants as an effective lattice where each site is labeled by partition of some integer. Due to the conservation of total momentum the Hamiltonian of the chiral Tomonaga-Luttinger model is block-diagonal in the total momentum index N and the linear dispersion implies that energies of all the states inside one block are degenerate and equal $N\Delta$. The interactions lift this degeneracy mixing the Slater determinants into coherent superpositions which are described by the bosonic quantum numbers. Thus the bosonic eigenstates of the full interacting model can be viewed as a generalization of the extended Bloch states in the tight-binding models with degenerate on-site energies.

The linearization of the dispersion is the approximation which allows to solve the problem of interacting electrons in one dimension. The deviations from the linearity mixes the bosonic states, so they cannot any longer provide the solution to the model. In this work we introduce the random dispersion into the Tomonaga-Luttinger model and study the properties of its eigenstates. It is expected to model a system where the single particle spectrum is complicated. The question we address is whether this model exhibits localization, since the randomness introduced into the dispersion leads to randomness of the diagonal matrix elements of the Hamiltonian analogous to the diagonal disorder introduced into tight-binding Anderson models. We have found that the states remain localized in the basis of Slater determinants, i.e. resemble the states of the non-interacting systems.

The outline of this paper is as follows. In section 2 we review some known facts about the standard Tomonaga-Luttinger model. We describe the structure of the Hilbert space as an effective lattice and discuss the bosonization solution in this context. In section 3 we introduce randomness into the dispersion law of the Tomonaga-Luttinger model and simplify the interaction matrix. Then we solve the resulting model with assumption of independence of the on-site energies. Section 4 is devoted to the study of relevance of the correlations between the energies. In the last section the results are discussed and further investigations are proposed.

II. HILBERT SPACE OF TOMONAGA-LUTTINGER MODEL

In this section we review a model exactly solvable for any interaction strength. The simplest model of interacting fermions is the one-branch (chiral) Tomonaga-Luttinger model [13]. Let n label the one-particle orbital with convention that in the ground state $|0\rangle$ of the non-interacting model, $n = 1$ denotes the first unoccupied state. Since a very large number of states is occupied (the number of fermions is large) the number n can be allowed to run from $-\infty$ to $+\infty$. Another consequence is that the one-particle spectrum is assumed to be equidistant: $E_n = n\Delta$. Both the linear dispersion and the fact that the spectrum is unbound are crucial for the solubility of the model. Introducing creation and annihilation operators $c_n^\dagger$, c_n for a fermion on the orbital n, with usual anti-commutation relations $\{c_n, c_{n'}^\dagger\} = \delta_{n,n'}$ (other anti-commutators vanish) the Hamiltonian of the problem can be written as

$$\hat{H} = \hat{H}_0 + \hat{U} = E_0 + \sum_{n=-\infty}^{+\infty} E_n \ :c_n^\dagger c_n: + \sum_{m=1}^{+\infty} \sum_{n,n'=-\infty}^{+\infty} V_m c_{n+m}^\dagger c_n c_{n'-m}^\dagger c_{n'} \tag{1}$$

The normal ordering of operators having infinite expectation values is defined as $:\hat{O} := \hat{O} - \langle 0|\hat{O}|0\rangle$. As a result the constant E_0 absorbs the (infinite) free ground state energy together with the Hartree term:

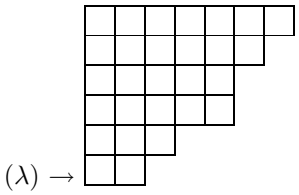
$$E_0 = \langle 0|\hat{H}|0\rangle = \sum_{n\le 0} E_n \langle 0|c_n^\dagger c_n|0\rangle + \frac{V_0}{2} \sum_{n,n'\le 0} \langle 0|c_n^\dagger c_n c_{n'}^\dagger c_{n'}|0\rangle \tag{2}$$

The operator of "total momentum"

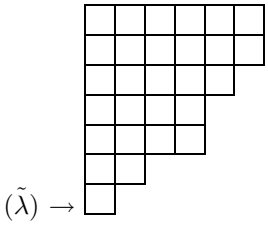
$$\hat{P} = \sum_{n=-\infty}^{+\infty} n \ :c_n^\dagger c_n: \tag{3}$$

commutes with the full Hamiltonian. Due to the linear dispersion this operator is proportional to the free part of the Hamiltonian, i.e. the first term on the RHS of (1).

Instead of solving this model by the standard method of bosonization using identities among second quantized operators (see for example [13]) we prefer to reformulate our interacting problem in order to relate it to the problems usually discussed within the framework of Anderson localization. We start by labeling the eigenstates of non-interacting problem, which are the familiar Slater determinants, by shifts of particles with respect to the non-interacting ground state as shown in the Fig. 1: the uppermost particle is shifted up λ_1 levels, the second one is shifted up λ_2 levels etc. The numbers λ_i are defined to satisfy $\lambda_i \geq \lambda_{i+1}$ for each i. If a particle is not shifted it is not recorded. The set (λ) of positive integers satisfying $\sum \lambda_i = N$ forms the set of partitions of N and is conveniently displayed by a Young diagram — array with λ_i boxes in the i-th row. Following the standard notations in the mathematical literature the numbers λ_i will be referred to as *parts* in what follows. For example, the partition of $N = 28$, $(\lambda) = (7, 6, 5, 5, 3, 2)$ sometimes written as $(7, 6, 5^2, 3, 2)$ that is displayed in Fig. 1 is represented graphically as



This pictorial representation of Slater determinants has nice properties under particle-hole interchange: recording shifts of the holes (which are moved downwards in energy) rather than particles in Fig. 1 yields the partition $(\tilde{\lambda}) = (6, 6, 5, 4, 4, 2, 1)$ whose graphical representation is just the transpose of the Young diagram for (λ):



Partition $(\tilde{\lambda})$ is said to be conjugate or dual to (λ). This duality results from the fact that each move of a particle upwards is accompanied by a move of a hole downwards.

The eigenstates of the non-interacting Hamiltonian labeled by partitions of N are degenerate and their energy is given by

$$\sum_i (E_{1-i+\lambda_i} - E_{1-i}) = \Delta \sum_i \lambda_i = N\Delta \tag{4}$$

The interactions conserve this number. The Hamiltonian (1) is therefore block-diagonal and does not mix the partitions of different N so we can restrict ourselves to the subspace of the Hilbert space spanned by partitions of N.

The dimensionality of this subspace is given by $p(N)$ — the number of partitions of N. There is no closed analytical expression for this number but the Euler expression [14] for the generating function

$$p(q) = \sum_{N=1}^{\infty} p(N)\, q^N = \prod_{l=1}^{\infty} \frac{1}{1-q^l} \tag{5}$$

allows to obtain the asymptotic behavior of $p(N)$ for large N and it is given by the formula of Hardy-Ramunajan:

$$p(N) \sim \frac{e^{\pi\sqrt{2N/3}}}{4\sqrt{3}N} \tag{6}$$

It demonstrates that the number of "sites" in the subspace increases rapidly with N.

Since for a given subspace N the free part of the Hamiltonian is proportional to the unit matrix the solution consists in diagonalization of the interacting part U. The interaction term in (1) is conveniently represented in our picture as effective binding between the sites. In order to present the results in a compact and transparent way we drop the requirement on a partition labeling the basis state to be represented by a non-increasing sequence. Instead we extend the notion to any finite sequence of integers (either positive or negative or zero) and use the following rules to relate this sequence to the standard (non-increasing) partition:

1. *In any sequence* $(\lambda_1, \lambda_2, \ldots)$ *two consecutive parts may be interchanged provided that the preceding part is decreased by unity and the succeeding part increased by unity, the state vector which corresponds to this partition acquires a minus sign*, i.e.

$$|(\lambda_1, \ldots, \lambda_i, \lambda_{i+1}, \ldots)\rangle = -|(\lambda_1, \ldots, \lambda_{i+1} - 1, \lambda_i + 1, \ldots)\rangle \tag{7}$$

2. *If any part exceeds by unity the preceding part the partition corresponds to zero vector*, i.e.

$$|(\lambda_1, \ldots, \lambda_i, \lambda_i + 1, \ldots)\rangle = 0 \tag{8}$$

3. *If the last part of the sequence is negative the partition corresponds to the zero vector.*

These rules, used in the theory of symmetric functions [15] are in fact nothing but a standard properties of Slater determinants under action of second quantized operators. Rule 1 results from the fact that i-th particle is excited to the level n_i such that

$$n_i = \lambda_i + 1 - i$$

with the energy $E_i = n_i\Delta$ while the $(i+1)$-th particle is excited to the level

$$n_{i+1} = \lambda_{i+1} - i$$

and interchanging consecutive rows in a (Slater) determinant results in a sign change. Rule 2 results from the fact that $\lambda_{i+1} = \lambda_i + 1$ implies $n_{i+1} = n_i$ which contradicts the Pauli principle. Rule 3 reflects the fact that negative λ_i describe (forbidden) transitions to occupied states inside the Fermi sea.

Let us denote by $\eta_{(\lambda)}$ the factors $-1, 0$ or $+1$ acquired when bringing a partition (λ) to its standard form. In these notations the matrix element of the interactions between the partitions (λ) and (μ) can be represented as

$$U_{(\lambda),(\mu)} = \langle(\lambda)|\hat{U}|(\mu)\rangle = \sum_m V_m \sum_{i,j} \langle(\ldots \lambda_i - m \ldots)|(\ldots \mu_j - m \ldots)\rangle. \tag{9}$$

Since the basis of Slater determinants is orthonormal the following expression is obtained

$$U_{(\lambda),(\mu)} = \sum_m V_m \sum_{i,j} \eta_{(\ldots\lambda_i - m\ldots)}\eta_{(\ldots\mu_j - m\ldots)}\delta_{(\ldots\lambda_i - m\ldots),(\ldots\mu_j - m\ldots)}. \tag{10}$$

The eigenstates of U are

$$|(l)\rangle = \sum_{(\lambda)} \chi^{(\lambda)}_{(l)}|(\lambda)\rangle \tag{11}$$

where $\chi^{(\lambda)}_{(l)}$ are the characters of the irreducible representation (λ) of an element of the symmetric group S_N belonging to the class (l). For this purpose it is shown in Appendix A that

$$\sum_{i,j} \eta_{(\ldots\lambda_i - m\ldots)}\eta_{(\ldots\lambda_i - m\ldots\lambda_j + m\ldots)}\chi^{(\ldots\lambda_i - m\ldots\lambda_j + m\ldots)}_{(l)} = m n_m \chi^{(\lambda)}_{(l)} \tag{12}$$

where n_m is the number of times the cycle m appears in the class (l). This leads to

$$\hat{U}|(l)\rangle = \sum_m V_m m n_m \sum_{(\lambda)} \chi^{(\lambda)}_{(l)}|(\lambda)\rangle = \left(\sum_m V_m m n_m\right)|(l)\rangle. \tag{13}$$

The state $|(l)\rangle$ is therefore an eigenstate of the Hamiltonian (1) with the eigenvalue

$$E_{(l)} = E_0 + N\Delta + \sum_m V_m m n_m. \tag{14}$$

The orthogonality of the characters implies that the states $|(l)\rangle$ form an orthogonal basis, namely:

$$\langle (l)|(l')\rangle = \frac{g}{g_{(l)}}\,\delta_{(l),(l')} \tag{15}$$

where $g = N!$ is the total number of elements in S_N and the number of elements in the class (l) is

$$g_{(l)} = N! \prod_{m=1}^{\infty} \frac{1}{m^{n_m} n_m!}. \tag{16}$$

We normalize the states $|(l)\rangle$ as

$$|(l)\rangle = \sqrt{\frac{g_{(l)}}{g}} \sum_{(\lambda)} \chi_{(l)}^{(\lambda)} |(\lambda)\rangle \tag{17}$$

so that they form an orthonormal basis. These are standard bosonic states usually labeled by the occupation numbers n_m that are the eigenstates of the interacting Hamiltonian (1). The transformation with the matrix

$$\langle (\lambda)|(l)\rangle = \sqrt{\frac{g_{(l)}}{g}} \chi_{(l)}^{(\lambda)} \tag{18}$$

is the unitary transformation of the basis vectors in our Hilbert space, which diagonalizes the interacting Hamiltonian (1). When the free part of the Hamiltonian is degenerate in the subspace N the states $|(l)\rangle$ play the role of the extended states (in the Hilbert space). In order to verify this statement we calculated numerically the inverse participation ratio P, defined as

$$P = \sum_{(\lambda)} |\langle (\lambda)|(l)\rangle|^4 \tag{19}$$

for some arbitrary states (l). Although no closed expression is available for the general amplitude (18) of the state $|(l)$ on the site (λ), the beautiful recurrent relation of the characters of the symmetric group (A12) (based on the Frobenius formula) was implemented numerically to calculate these amplitudes. The inverse participation ratio averaged over the bosonic states (l) is shown in Fig. 2 for different values of N. As the number of sites $N_{site} = p(N)$ increases the inverse participation ratio P was found to decay as $P = 1/N_{site}^{\alpha}$, where $\alpha \approx 0.8$. This allows us to conclude that the bosonic states (l) are indeed extended in the basis of free Slater determinants (λ).

III. SIMPLIFIED INTERACTIONS

The existence of extended states in the chiral Tomonaga-Luttinger model can be attributed to the degeneracy of the diagonal elements of the Hamiltonian (1), since the interactions (10) have short-range in the space of partitions. In order to define more precisely what we mean by short-range in our Hilbert space we have to order the partitions in some order. Then the interaction can be shown to connect only blocks of elements that are not too far one from the other.

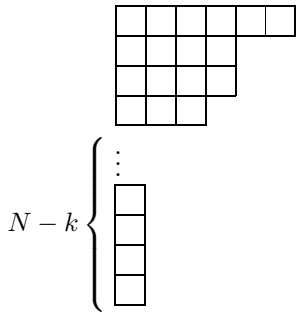
It is natural to order the partitions in the lexicographic order, i.e. if $\lambda_i = 1, 2, 3 \ldots$ numbers the letters in an alphabet the lexicographic order of partitions corresponds to the order of words in Arabic or Hebrew dictionary. For example, the 11 partitions of $N = 6$ in the lexicographic order are:

$$\begin{aligned}
k = 1 \quad & (1,1,1,1,1,1) \\
k = 2 \quad & (2,1,1,1,1) \\
k = 3 \quad & (3,1,1,1) \\
k = 4 \quad & (2,2,1,1) \\
& (4,1,1) \\
k = 5 \quad & (3,2,1) \\
& (5,1) \\
k = 6 \quad & (2,2,2) \\
& (4,2) \\
& (3,3) \\
& (6)
\end{aligned} \tag{20}$$

The partitions are grouped by k—the number of 1's (that appear always on the right). The k-th group contains partitions of the form

$$(\lambda_1, \lambda_2, \dots \lambda_l, 1^{N-k}) \tag{21}$$

with constraint $\lambda_i \geq 2$, with the only exception for the first group containing (1^N). The typical Young diagram of a partition in the k-th group is of the form



(22)

To calculate the number of partitions in the k-th group we note that exactly $p(k)$ partitions have at least $N-k$ ones. Among them $p(k-1)$ have at least $N-(k-1)$ ones and the rest $n(k) = p(k) - p(k-1)$ have *exactly* $N-k$ ones. Therefore the k-th group contains $n(k)$ partitions.

Usually the matrix element of the interaction potential V_m is a decreasing function of the momentum transfer m, with some effective range m_c. In what follows it will be convenient to analyze a model where the only non-zero matrix element corresponds to $m = 1$ with $V_1 = V$. It will not affect qualitatively our results, since we are primarily interested in the limit of large N subspace. The matrix element of this interaction is given by the simplified version of (10)

$$U_{(\lambda),(\mu)} = V \sum_{i,j} \delta_{(\dots \lambda_i - 1 \dots),(\dots \mu_j - 1 \dots)}. \tag{23}$$

The factors η are omitted since the partitions $(\dots \lambda_i - 1 \dots)$ and $(\dots \mu_j - 1 \dots)$ are either already in the standard (non-increasing) form or corresponds to zero vector and do not contribute to the sum. Therefore the interaction couples partitions (λ) and (μ) that differ in two of the λ_i's, namely $\lambda_i - \mu_i = 1$ and $\lambda_j - \mu_j = -1$. The Young diagram of the partition (λ) is obtained from the one of the partition (μ) by moving one square under the constraint that the new Young diagram is legitimate. Consider the k-th group. If $\mu_i > 1$ and $\mu_j > 2$ the coupling is between partitions inside this group. This corresponds to moving a square between $\mu_i > 1$ rows of the Young diagram. In order to couple to neighboring groups a square corresponding to $\mu_i = 1$ should be moved. Moving a square from/to any row with $\mu_j > 1$ to/from the region where $\mu_i = 1$ leads to a coupling to the neighboring group $k \pm 1$. The exception is if a square is moved from $\mu_i = 2$ (that has only two squares and after the move only one is left) and is attached as the last square in the tail of $\mu_j = 1$, or if the last square is moved to $\mu_j = 1$ turning it into a $\mu_j = 2$ row. In this case the length of the tail of $\mu_j = 1$ is changed by two, i.e. the coupling is to the group $k \pm 2$. These are the only moves of one square, therefore only the groups k' such that $|k - k'| \leq 2$ are coupled to the k-th group and the interaction between the various blocks of partitions is of short range. Fig. 3 shows the structure of the interaction matrix for $N = 14$.

It is therefore natural to expect that the diagonal disorder will destroy the extended states of the chiral Tomonaga-Luttinger model. In the original model in the absence of interactions the eigenenergies $E_{(\lambda)}$ are independent of the partition and take the constant value $E_{(\lambda)} = E_0 + N\Delta$. We introduce a modification and assume that the $E_{(\lambda)}$ are independent random variables. This assumption is made for the sake of simplicity. It cannot be correct since there are $p(N)$ values of $E_{(\lambda)}$ that depend on $2N$ level spacings. An assumption that is more physical will be introduced in the following section.

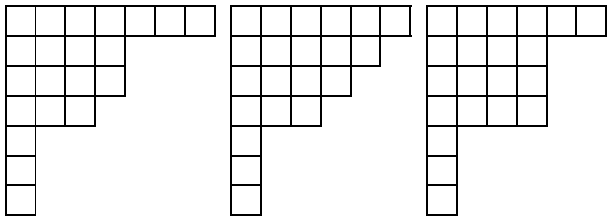
In order to be able to solve the model with random energies we simplify further the interaction (23). From the experience in research of Anderson localization we expect that the behavior of the eigenvectors will not be affected by the precise form of the short-range off-diagonal matrix elements, so we make the following simplifications. We assume that the effect of transitions inside the same group of partitions is to delocalize the states within the group. The localization within a group is expected only to enhance the overall localization. Next, we neglect the coupling to the $k+2$-th group. The reason is that for a given partition (λ) there exists only one coupling term of this kind. It couples

$$\left((\dots), 1^{N-k}\right) \quad \text{and} \quad \left((\dots)', 2, 1^{N-k-2}\right) \tag{24}$$

This should be compared it with the couplings of (λ) to the next group:

$$\left((\ldots), 1^{N-k}\right) \to \left((\ldots)', 1^{N-k-1}\right) \tag{25}$$

The number of such couplings is given by the number of different $\lambda_i \geq 2$ in the partition (λ), which is generally much larger than 1 for large enough k. It is equal to the number of rows with $\lambda_i \geq 2$ in the Young diagram of the partition. Consider for example the partition $(6, 4^2, 3, 1^4)$ of $N = 21$ belonging to he the 17-th group corresponding to the Young diagram (22). It has (apart of ones) three different parts: 6,4,3 and is coupled to 3 partitions in the 18-th group:



$(7, 4^2, 3, 1^3)$ $(6, 5, 4, 3, 1^3)$ $(6, 4^3, 1^3)$ (26)

Another way to visualize the number of coupling is to count the number of concave corners in the Young diagram that is equal to the number of parts namely lines of different length larger than unity.

In Appendix B we show that $d(k)$ — the number of different parts in all partitions of k with $\lambda_i \geq 2$ is exactly $p(k-2)$ — the number of (any) partitions of $(k-2)$. Therefore the mean number of transitions from the k-th group to the $k+1$-th group is given by $t(k) = d(k)/n(k) = p(k-2)/\left(p(k) - p(k-1)\right)$.

The Schrödinger equation for our simplified model can be written as a tight-binding form where the groups of partitions are the effective sites:

$$(E - E_{(\lambda)})\psi(\lambda)_k = \sum_{(\mu)} U_{(\lambda),(\mu)}\psi(\mu)_{k-1} + \sum_{(\nu)} U_{(\lambda),(\nu)}\psi(\nu)_{k+1} \tag{27}$$

where $\psi(\lambda)_k$ denotes the value of the wave function on site(λ) in the k-th group. Let us assume that the wave function is extended within the kth group. We introduce a simpler model that is expected to exhibit similar localization properties [17]. We replace the interaction $U_{(\lambda),(\mu)}$ by an average interaction that couples each partition in the k-th group to all partitions in the group $k \pm 1$,

$$V_k = \frac{\sum_{(\nu)} U_{(\lambda),(\nu)}}{n(k+1)} \tag{28}$$

In this approximation (27) reduces to

$$(E - E_{(\lambda)})\psi(\lambda)_k = V_{k-1}\sum_{(\mu)} \psi(\mu)_{k-1} + V_k \sum_{(\nu)} \psi(\nu)_{k+1} \tag{29}$$

The numerator in (28) can be estimated as V times the mean number of couplings with partitions in $(k+1)$-th group:

$$V_k = V\frac{t(k)}{n(k+1)} = V\frac{d(k)}{n(k)n(k+1)} \tag{30}$$

Denoting the sum of the wave-functions over a group by $C_k = \sum_{(\lambda)} \psi(\lambda)_k$ we find with the help of (29) that

$$\mathcal{E}_k C_k = V_{k-1}C_{k-1} + V_k C_{k+1} \tag{31}$$

where

$$\frac{1}{\mathcal{E}_k} = \sum_{(\lambda)} \frac{1}{E - E_{(\lambda)}} \tag{32}$$

and the sum is over all partitions in the group k.

For random independent $E_{(\lambda)}$ all the $\mathcal{E}_k$ are independent random variables and the tight-binding model (31) is an Anderson model at zero energy. In the spirit of transfer matrix method it is expected that also for $\mathcal{E}_k$ given by (32)

similar localization properties will be found. The properties of the solution of this one-dimensional model can be analyzed within the framework of Anderson localization in one-dimensional systems [20].

One can take the uniform distribution of energies that turns out to be convenient for calculations. Remembering that each many-body energy is a sum of N level spacings we assume the following distribution:

$$P(E) = \begin{cases} 1/\sqrt{N}\Delta, & -\sqrt{N}\Delta/2 < E < \sqrt{N}\Delta/2 \\ 0, & \text{otherwise} \end{cases} \tag{33}$$

For a variety of distributions the distribution of $\mathcal{E}_k$ tends to the Cauchy distribution:

$$P(\mathcal{E}_k) = \frac{1}{\pi} \frac{\delta_k(E)}{\mathcal{E}_k^2 + \delta_k(E)^2} \tag{34}$$

where $\delta_k(E) = \delta(E)/n(k)$ and for (33) $\delta(E) = \sqrt{N}\Delta/\pi$.

Multiplying the equation (31) by $n(k)n(k+1)/(Vd(k))$ and observing that in the limit $k \to \infty$ one finds $n(k)/n(k+1) \to 1$ and $d(k)/d(k-1) \to 1$ so that (31) takes the form

$$\mathcal{E}_k C_k = C_{k-1} + C_{k+1} \tag{35}$$

It is a Lloyd model with a k-dependent distribution of on-site energies characterized by parameter

$$\delta_k(E) = \frac{\delta(E)}{V} \frac{n(k+1)}{d(k)} \tag{36}$$

which in the limit $k \to \infty$ has the property

$$\delta_k(E) \sim \sqrt{\frac{N}{6k}} \frac{\Delta}{V} \tag{37}$$

We generalize the solution of the Lloyd model [20] to the present case of site-dependent distribution of energies. We observe that the distribution parameter depends weakly on the site number k and use the site-dependent localization length ansatz:

$$1/\xi_k = \ln \frac{|C_k|}{|C_{k+1}|} = \cosh^{-1} \sqrt{1 + \frac{\delta_k^2}{4}} \approx \ln \delta_k = \ln \sqrt{\frac{N}{6k}} \frac{\Delta}{V} \tag{38}$$

The last approximation is justified in the weak-coupling limit $V/\Delta \ll 1$.

The validity of the site-dependent localization length assumption is checked numerically. The on-site energies $\mathcal{E}_k$ were generated from the Cauchy distribution (34) with parameter δ_k given by (37). The inverse localization length $1/\xi$ (Lyapunov exponent) was calculated using the standard technique of transfer matrices [20] and it was found that the site-dependent ansatz (38) describes well (at least qualitatively) the behavior of the localization length. Thus for independently distributed energies the typical behavior of the wave functions is described by coefficients that decay as

$$C_k \sim e^{-k/\xi_k} = \exp\left(-k \log \sqrt{\frac{N}{6k}} \frac{\Delta}{V}\right). \tag{39}$$

In the strong coupling limit similar considerations lead to

$$C_k \sim e^{-k/\xi_k} = \exp\left(-k \sqrt{\frac{N}{24k}} \frac{\Delta}{V}\right), \tag{40}$$

but in this limit these require some further justification. The importance of this result is that numerical studies of more realistic many-particle energy distributions confirm the validity of (39). This will be the subject of the next section.

IV. EFFECTS OF ENERGY CORRELATIONS

In the last section we considered a model in which the diagonal matrix elements $E_{(\lambda)}$ are independent random variables. We did not specify the distribution law of these energies, since the solution does not depend on fine details, but rather on general properties of the distribution. We would like to stress that the assumption of statistical independence of the energies $E_{(\lambda)}$ is not realistic for a many-body system. To illustrate this statement a simple argument can be given. Suppose we are dealing with a disordered mesoscopic system and the one-particle spectrum is random, i.e. consists of levels with independently distributed spacings $\Delta_j = E_j - E_{j-1}$. The generalization of the expression (4) for the energy of the partition (λ) for this case is

$$E_{(\lambda)} = \sum_i (E_{1-i+\lambda_i} - E_{1-i}) = \sum_i \sum_{j=2-i}^{\lambda_i+1-i} \Delta_j \tag{41}$$

In order to describe the energies $E_{(\lambda)}$ in the subspace N we need $2N$ random quantities Δ_j. On the other hand the number of states in this subspace is given by $p(N)$, the number of partitions of N which grows exponentially for $N \to \infty$. Therefore there exist much more many-body energies $E_{(\lambda)}$ than independent level spacings. Therefore the many-particle energies $E_{(\lambda)}$, which are the sums of the one-particle energies, are statistically dependent.

The analytical calculation of the (joint) distribution of the parameters $\mathcal{E}_k$ defined in (32) for the effective tight-binding model (31) seems to be hopeless. Nevertheless we expect some features of this distribution, based on the formula (32). For an eigenstate the eigenvalue E is in the interval $[-1 + E^{min}_{(\lambda)}, +1 + E^{max}_{(\lambda)}]$, where $E^{min}_{(\lambda)}$ and $E^{max}_{(\lambda)}$ are the minimal and maximal values of $E_{(\lambda)}$. The value of E can be very close one of the $E_{(\lambda)}$. Therefore the terms in the sum on the RHS of (32) can be positive or negative and they strongly fluctuate in magnitude. Therefore there is a finite probability that this sum will take a value in the vicinity of zero. Consequently it is expected that the distribution $P(\mathcal{E}_k)$ is broad with a diverging second moment or variance:

$$\Delta\mathcal{E}_k^2 = \langle(\mathcal{E}_k - \langle\mathcal{E}_k\rangle)^2\rangle \to \infty \tag{42}$$

Another important issue is the effective statistical independence of $\mathcal{E}_k$ for different k-s. It is known for Anderson localization that the long-range correlations between the on-site energies change dramatically the criteria for the onset of the localization (see [18] and references therein). The measure of statistical dependence of energies in our case is the correlation matrix defined as

$$C_{kl} = \frac{\langle(\mathcal{E}_k - \langle\mathcal{E}_k\rangle)(\mathcal{E}_l - \langle\mathcal{E}_l\rangle)\rangle}{\Delta\mathcal{E}_k \Delta\mathcal{E}_l} \tag{43}$$

Because of the strong fluctuations between the terms of the sum on the RHS of (32), that are different for the various groups, it is reasonable that the off-diagonal terms of the correlation function are much smaller than the diagonal ones. Usually for localization only pair correlations are important [19]. In the view of the diverging variance we assume that $C_{kl} = \delta_{kl}$, an assumption that will be tested numerically.

These facts were checked numerically and in the approximation of independent energies the effective distribution $P(\mathcal{E}_k)$ can be calculated. For this purpose $M = 1000$ realizations of $N = 30$ level spacings Δ_j were generated from the exponential distribution:

$$P(\Delta) = e^{-\Delta} \tag{44}$$

with $\bar{\Delta} = 1$. Then we generated numerically the whole list of $p(30) = 5604$ partitions of $N = 30$ and for each realization of level spacings the energies $E_{(\lambda)}$ were calculated using (41). Out of these energies the effective energies $\mathcal{E}_k$ were obtained using the definition (32) for $E = 0$. Several statistical tests were performed and results are shown in Fig. 4-5.

The correlation matrix of the energies $\mathcal{E}_k$ was computed. The $\langle\ldots\rangle$ averages were performed over realizations of the level spacings. The result is shown in Fig. 4. It is plausible that the correlations are of short range in the space of indices k and only the diagonal elements of C_{kl} are of appreciable magnitude. This behavior can be attributed to the diverging variance.

Due to the fact that the distribution of the energies $\mathcal{E}_k$ is expected to have a diverging variance the distribution $P(\mathcal{E}_k)$ cannot be calculated from a histogram. Instead its characteristic function

$$X_k(p) = \langle e^{ip\mathcal{E}_k}\rangle \tag{45}$$

was computed and plotted for several values of k in Fig. 5 as a function of p. The diverging second moment due to the fact that $P(x) \propto 1/x^2$ manifests itself in the discontinuity of the derivative at $p = 0$ as can be seen from the Fig 6. Therefore we have strong evidence that the energies $\mathcal{E}_k$ have a broad distribution.

Observing the behavior of the characteristic function shown on the Fig. 5 and Fig. 6 we introduced a scaling ansatz for the distribution function:

$$P(\mathcal{E}_k) = P_k(\mathcal{E}) = \frac{n(k)}{\Delta} \widetilde{P}\left(n(k)\frac{\mathcal{E}}{\Delta}\right). \tag{46}$$

The universal function $\widetilde{P}(x)$ is independent of k. The scaling of the distribution implies that the characteristic function (45) satisfies

$$X_k(p) = \widetilde{X}\left(\frac{p\Delta}{n(k)}\right), \qquad \widetilde{X}(q) = \int_{-\infty}^{\infty} dx\, \widetilde{P}(x) e^{iqx} \tag{47}$$

which is easy to check by plotting $X_k(p)$ as a function of $q = p\Delta/n(k)$ for different values of k. These curves are displayed in Fig. 7 and are indeed found to be close to each other in spite of the fact that $n(k)$ ranges from $n(1) = 1$ to $n(30) = 1039$.

We found numerically that the universal characteristic function can be well approximated by the general form

$$\widetilde{X}(q) = \exp\left(-a|q| - \frac{bq^2}{2}\right) \tag{48}$$

The value of parameters were found to be $a \approx 1.15$, $b \approx 6.32$. This is the characteristic function of a distribution of a sum of two random variables x_1 and x_2, where x_1 is drawn from Cauchy distribution with probability density

$$p_1(x) = \frac{1}{\pi}\frac{a}{x^2 + a^2} \tag{49}$$

and the second is distributed normally according to

$$p_2(x) = \frac{1}{\sqrt{2\pi b}} e^{-\frac{x^2}{2b}}. \tag{50}$$

The distribution of $x = x_1 + x_2$ is thus given by

$$\tilde{P}(x) = \int_{-\infty}^{+\infty} dy p_1(x - y) p_2(y) \tag{51}$$

and its characteristic function is (48). It has a diverging variance due to the fact that $\tilde{P}(x)$ behaves as $\frac{1}{\pi}\frac{a}{x^2}$ for large $|x|$.

The scaling factor $\Delta/n(k)$ and the limiting behavior of the distribution function are the same as those found in the case of mutually independent energies. This allows us to conclude that the behavior of the coefficients C_k given by (39), that was obtained in the model with uncorrelated $E_{(\lambda)}$, is satisfied also by the eigenfunctions of the more realistic model of the present section.

V. DISCUSSION

The localization properties of the eigenstates of the modified interacting chiral Tomonaga-Luttinger model were studied. For total energy $N\Delta$ subspace we defined an effective lattice of Slater determinants labeled by partitions of N which represent the eigenstates of the system in the absence of interactions. We have observed that in the case of linear dispersion the eigenstates of the interacting model described by the bosons are extended over this lattice. When the random one-particle levels are introduced instead of the equidistant spectrum, the bosons are no more the eigenstates and the problem of diagonalizing of the Hamiltonian. In order to achieve this goal we have grouped the different sites (partitions) according to the number of ones, representing the distance of the most excited particle from the Fermi surface in the original Slater determinant. This is to be contrasted with the grouping of many-body states into generations according to the number of excited particles and holes as was firstly done in [6]. Each resulting group of partitions was given a label k. The interactions were shown to be of short range in k. The interactions

were further simplified by assuming that they interconnect only groups of partitions corresponding to the adjacent values of the index k (and the interactions between groups k and $k \pm 2$ were neglected). Assuming further that the states are extended within each group k we obtained an effective tight-binding Anderson model where each group k has an effective random energy and the hopping takes place between the nearest neighbors only. In the assumption of independently distributed on-site energies the model is found to be close to the Lloyd model characterized by a broad distribution of the on-site energies. The main feature of our effective model is that the distribution of energies is k-dependent. We employed the site-dependent localization length assumption, which consists in assuming the parameters of distribution change slowly, so locally the localization length is given by the same formula as for the Lloyd model with parameters of the local distribution instead of the uniform global one. Up to an unknown numerical factor this solution describes well the behavior of the localization length as a function of k and it enabled us to show that the eigenstates are localized in the space of groups of partitions, k. The weak-coupling and the strong-coupling regimes can be identified according to the value of the parameter V/Δ. In the weak-coupling regime $V \ll \Delta$, the amplitudes decay as $\exp\left(-k \log \sqrt{\frac{N}{6k}} \frac{\Delta}{V}\right)$, namely nearly exponentially. In the strong coupling regime $V \gg \Delta$, this behavior changes to $\exp\left(-\sqrt{\frac{Nk}{24}} \frac{\Delta}{V}\right)$.

One important point which was discussed is the effect of correlations between the many-body energies of the non-interacting system. In the studies devoted to the localization in many-body systems the effective on-site energies corresponding to the eigenstates of the non-interacting system are assumed to be independent random variables, so their spectrum is described by a Poissonian sequence. This is not the case for the true many-body fermionic system, since these energies are sums of independent energies of one-particle excitations. The number of independent random variables is therefore much less than the number of different many-body states. This situation is similar in some sense to one faced in random walk problems, where the increments are distributed independently, while there is a strong correlation between the positions of the random walk at different times. In our work we considered this problem numerically and found that although the correlation between the exact energies corresponding to the eigenstates of the free model are not *a priori* negligible the random energies of the effective sites k are found to be very weakly correlated. Moreover it was found that the parameters of the distribution of the effective energy of the group k scale with the number of the partitions inside the group in the same way as they do in the case of independent many-body energies. This led us to the conclusion that our result for the localization of the eigenstates are valid for more realistic assumptions on the distribution of the many-body energies.

The main open problem arising from the present discussion is the study of the interacting electrons in dimensions higher than one by the methods of high-dimensional bosonization. In this technique [24–28] the fermionic system is described as an infinite set of one-dimensional Luttinger models attached to a point on the Fermi surface. Therefore the interactions mix the states belonging to different points in addition to the coupling within the same point (represented by a Luttinger model), which can be treated within the framework developed in the present paper. This possibly can be a mechanism for the delocalization of the states and a resulting localization transition must be compared with the transition encountered in the recent studies [6,8–10] of interacting electrons in finite systems.

Another question to be explored in the future is the nature of localization in the Hilbert space for single-particle dispersions that are more realistic than the random ones.

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APPENDIX A: A PROPERTY FOR CHARACTERS OF S_N

In this appendix we show that the property (12)

$$\sum_{i,j} \eta_{(\ldots\lambda_i - m\ldots)} \eta_{(\ldots\lambda_i - m\ldots\lambda_j + m\ldots)} \chi_{(l)}^{(\ldots\lambda_i - m\ldots\lambda_j + m\ldots)} = m n_m \chi_{(l)}^{(\lambda)}$$

is satisfied for the characters of the symmetric group S_N. We did not find this property in any textbook on the representations of groups, but its validity follows from the known properties of the characters of S_N found, for example, in [15] and [16]. We present the proof of this property here for the sake of completeness of the discussion.

The Frobenius formula as it is written in the classic textbooks on group theory [16,15] states the following identity between two antisymmetric functions of N-dimensional vector $z = (z_1, z_2, \ldots, z_N)$:

$$S_{(l)}(z)D(z) = \sum_{(\lambda)} \chi^{(\lambda)}_{(l)} \Psi_{(\lambda)}(z) \tag{A1}$$

where $\chi^{(\lambda)}_{(l)}$ is the character of the class (l) of the symmetric group S_N in the irreducible representation (λ). With the partition (λ) one associates the totally antisymmetric function:

$$\Psi_{(\lambda)}(z) \equiv \sum_P \mathrm{sgn}(P)\, z_{P(1)}^{\lambda_1+N-1} z_{P(2)}^{\lambda_2+N-2} \ldots z_{P(N)}^{\lambda_N} \tag{A2}$$

where the sum runs over all the permutations of $1, 2, \ldots, N$, the sign of permutation being $\pm$ for an even/odd permutation. This is, up to a normalization constant, a Slater determinant constructed out of one-particle wavefunctions $z^{n_i} = z^{\lambda_i+N-i}$, where $z = e^{i\theta}$. The factor $D(z)$ in (A1) is the Vandermonde determinant:

$$D(z) = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_N \\ z_1^2 & z_2^2 & \cdots & z_N^2 \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_N^{N-1} \end{vmatrix} = \prod_{i<j} (z_i - z_j) \tag{A3}$$

representing the ground state of N fermions. Let S_m be a power sum of variables z_j defined as follows:

$$S_m(z) = \sum_j z_j^m \tag{A4}$$

The function $S_m(z)$ is totally symmetric function of the z_j-s. These functions are linearly independent as can be checked by calculating the Jacobian:

$$\left| \frac{\partial S_m}{\partial z_j} \right| = N! D(z). \tag{A5}$$

The class (l) is characterized by $n_1, n_2, \ldots, n_m, \ldots$ cycles of length $1, 2, \ldots, m, \ldots$. For this class the totally symmetric function $S_{(l)}$ is defined as a product of power sums

$$S_{(l)}(z) = S_1^{n_1} S_2^{n_2} \ldots S_N^{n_N} = \prod_{m=1}^{\infty} S_m^{n_m} \tag{A6}$$

Since for different m-s the power sums $S-m$ are linearly independent so are the monomials $S_{(l)}$ for different partitions (l).

The Frobenius formula (A1) can be inverted with help of the orthogonality relation of the characters:

$$\Psi_{(\lambda)}(z) = \frac{1}{g} \sum_{(l)} g_{(l)} \chi^{(\lambda)}_{(l)} S_{(l)}(z) D(z) \tag{A7}$$

where $g = N!$ is the order of the S_N and $g_{(l)}$ is the number of elements of S_N in the class (l) given by (16).

We turn now to develop a recurrence relation between the characters. For this purpose it is instructive to consider the product of the antisymmetric function $\Psi_{(\mu)}(z)$ and a power sum $S_m(z)$. Since the power sum is totally symmetric the summands can be rearranged in the order set by each permutation and one obtains

$$\begin{aligned} S_m(z)\Psi_{(\mu)}(z) &= \sum_P \mathrm{sgn}(P) \left(\sum_i z_{P(i)}^m \right) z_{P(1)}^{\mu_1+N-1} z_{P(2)}^{\mu_2+N-2} \ldots z_{P(N)}^{\mu_N} = \\ &= \sum_i \eta_{(\ldots\mu_i+m\ldots)} \Psi_{(\ldots\mu_i+m\ldots)}(z) \end{aligned} \tag{A8}$$

where $\eta_{(\ldots\mu_i+m\ldots)} = -1, 0$ or $+1$ is determined accordingly to the rules of Section 2.

Having established the relation between the Slater determinants $\Psi_{(\mu)}$ for various partitions we turn to develop a recurrence relation between the characters. Consider class (l) with at least one m-cycle. Let us denote by (l'_m) the class obtained by removing one m-cycle from (l). It is then clear that

$$S_{(l)} = S_m S_{(l'_m)} = \left(\sum_j z_j^m \right) S_{(l'_m)} \tag{A9}$$

The partition (l'_m) defines a class in the symmetry group S_{N-m}. The Frobenius formula (A1) for this class reads

$$S_{(l'_m)} D(z) = \sum_{(\mu')} \chi^{(\mu')}_{(l'_m)} \Psi_{(\mu')} \tag{A10}$$

while for (l) it can be rewritten with the help of (A8) in the form

$$\begin{aligned} S_{(l)} D(z) = S_m S_{(l'_m)} D(z) = \sum_{(\mu')} \chi^{(\mu')}_{(l'_m)} S_m \Psi_{(\mu')} = \\ = \sum_{(\mu')} \chi^{(\mu')}_{(l'_m)} \sum_i \eta_{(\ldots \mu'_i + m \ldots)} \Psi_{(\ldots \mu'_i + m \ldots)} \end{aligned} \tag{A11}$$

Comparing the coefficients of the $\Psi_{(\mu')}$ in (A11) with those in (A1) we find the recurrence relation of the characters

$$\chi^{(\lambda)}_{(l)} = \sum_i \eta_{(\ldots \lambda_i - m \ldots)} \chi^{(\ldots \lambda_i - m \ldots)}_{(l'_m)} \tag{A12}$$

This relation yields the characters of S_N in terms of the characters of the symmetric group S_{N-m}.

In order to derive (12) take some particular m and let each partition (l) contain m as its part $n_m^{(l)}$ times. For each (λ) consider the following sum over partitions:

$$F_{(\lambda)}(z) = \frac{1}{g} \sum_{(l)} g_{(l)} m n_m^{(l)} \chi^{(\lambda)}_{(l)} S_{(l)}(z) \tag{A13}$$

If (l) has no parts equal to m, $n_m^{(l)} = 0$ and (l) does not contribute to this sum. We want to factor out $S_m(z)$ from the sum and for this purpose we can write $S_{(l)}(z) = S_m(z) S_{(l'_m)}(z)$, so that the sum is over (l'_m). With the help of (16) we see that the expansion coefficients satisfy

$$\begin{aligned} g_{(l)} m n_m / g = m n_m \prod_{k=1}^{\infty} \frac{1}{k^{n_k} n_k!} = \frac{m n_m}{m^{n_m} n_m!} \prod_{k \neq m} \frac{1}{k^{n_k} n_k!} \\ = \frac{1}{m^{n_m - 1} (n_m - 1)!} \prod_{k \neq m} \frac{1}{k^{n_k} n_k!} = g_{(l'_m)} / g' \end{aligned} \tag{A14}$$

where $g' = (N-m)!$. The sum in (A13) takes the form

$$F_{(\lambda)}(z) = \left(\sum_j z_j^m \right) \frac{1}{g'} \sum_{(l'_m)} g_{(l'_m)} \chi^{(\lambda)}_{(l)} S_{(l'_m)}(z) \tag{A15}$$

Application of the recurrence relation (A12) to the characters in the sum together with the inverse Frobenius formula (A7) yields

$$\begin{aligned} F_{(\lambda)}(z) D(z) = S_m(z) \sum_i \eta_{(\ldots \lambda_i - m \ldots)} \frac{1}{g'} \sum_{(l'_m)} g_{(l'_m)} \chi^{(\ldots \lambda_i - m \ldots)}_{(l'_m)} S_{(l'_m)}(z) D(z) = \\ = \sum_i \eta_{(\ldots \lambda_i - m \ldots)} S_m(z) \Psi_{(\ldots \lambda_i - m \ldots)}(z) \end{aligned} \tag{A16}$$

The relation (A8) is applied to show that

$$F_{(\lambda)}(z)D(z) = \sum_{i,j} \eta_{(\ldots\lambda_i - m\ldots)}\eta_{(\ldots\lambda_i - m\ldots\lambda_j + m\ldots)}\Psi_{(\ldots\lambda_i - m\ldots\lambda_j + m\ldots)}(z) \tag{A17}$$

Application of the inverse Frobenius formula (A7) yields

$$F_{(\lambda)}(z) = \frac{1}{g}\sum_{ij}\sum_{(l)} \eta_{(\ldots\lambda_i - m\ldots)}\eta_{(\ldots\lambda_i - m\ldots\lambda_j + m\ldots)} g_{(l)} \chi_{(l)}^{(\ldots\lambda_i - m\ldots\lambda_j + m\ldots)} S_{(l)}(z) \tag{A18}$$

Making use of the linear independence of $S_{(l)}$ for different partitions (l) and equating the coefficients of the $S_{(l)}(z)$ with the corresponding coefficients in (A13) results in the required relation (12).

APPENDIX B: NUMBER OF DISTINCT PARTS OF PARTITIONS.

In this appendix we show that $d(N)$ – the number of distinct parts in the partitions of N with parts $\lambda_i \geq 2$ equals $p(N-2)$ – the number of unrestricted partitions of $N-2$. Together with the generating function (5) for the number of partitions let us introduce the generating function

$$\tilde{p}(z,q) = \sum_{N=1}^{\infty}\sum_{d=1}^{\infty} \tilde{p}(N,d) q^N z^d \tag{B1}$$

for the number $\tilde{p}(N,d)$ of partitions of N having entries that satisfy $\lambda_i \geq 2$ and having exactly d distinct parts. For example for $N=6$ and $d=2$ this number is $\tilde{p}(N,d)=1$, corresponding to the partition $(4,2)$ as can be seen from the list (20). The number of distinct parts $d(N)$ in all partitions of N with parts ≥ 2 is then given by

$$d(N) = \sum_{d=1}^{N} d\tilde{p}(N,d) \tag{B2}$$

It is generated by the following function

$$d(q) = \sum_{N=1}^{\infty} d(N) q^N = z\frac{d}{dz}\tilde{p}(z,q)\Big|_{z=1} \tag{B3}$$

The explicit expression for $\tilde{p}(z,q)$ can be found easily summing directly over all the appropriate partitions:

$$\tilde{p}(z,q) = \sum_{(\lambda),\lambda_i \geq 2} q^{|(\lambda)|} z^{d(\lambda)} \tag{B4}$$

where $d(\lambda)$ is the number of distinct parts in (λ) and $|(\lambda)| = \sum_i \lambda_i$. It is convenient to use the sets of "occupation numbers" $\{n_m\}$, where n_m – the number of times m appears in the partition (λ). Noting that $|(\lambda)| = \sum_i \lambda_i = \sum_m m n_m$ and $d(\lambda) = \sum_m \theta(n_m)$, where $\theta(x)$ is the Heavyside step function we rewrite (B4) as a sum over all the configurations $\{n_m\}$, with $m \geq 2$

$$\begin{aligned}\tilde{p}(z,q) &= \sum_{\{n_m\},m\geq 2} q^{\sum_{m=2}^{\infty} m n_m} z^{\sum_{m=2}^{\infty}\theta(n_m)} = \prod_{m=2}^{\infty}\sum_{n_m=0}^{\infty} q^{m n_m} z^{\theta(n_m)} = \\ &= \prod_{m=2}^{\infty}\left(1 + \frac{z q^m}{1-q^m}\right)\end{aligned} \tag{B5}$$

Differentiating this expression with respect to z at $z=1$ we get

$$d(q) = z\frac{d}{dz}\prod_{m=2}^{\infty}\left(1+\frac{zq^m}{1-q^m}\right)\Big|_{z=1} = q^2\prod_{m=1}^{\infty}\frac{1}{1-q^m} = q^2 p(q) \tag{B6}$$

where $p(q)$ is given by (5). The function $q^2 p(q)$ is the generating function for $p(N-2)$. The equality of the generating functions $d(q)$ and $q^2 p(q)$ implies the equality of the coefficients: $d(N) = p(N-2)$, i.e. the number of different parts in all partitions of N with entries ≥ 2 is given by the number of unrestricted partitions of $N-2$. Applying this argumentation to all partitions ending with $N-k$ ones yields $d(k) = p(k-2)$.

APPENDIX C: DISTRIBUTION OF HARMONIC SUM OF INDEPENDENT RANDOM VARIABLES.

We consider the harmonic mean X defined as

$$\frac{1}{X} = \frac{1}{N}\sum_{n=1}^{N}\frac{1}{x_n} \tag{C1}$$

of N independent random variables $x_1, x_2, \ldots x_N$ drawn from the common distribution with probability density $P(x)$. The only requirement we impose on $P(x)$ is that

$$\lim_{x\to 0} P(x) = C > 0 \tag{C2}$$

Consider the random variable $y = 1/x$. Its distribution function $Q(y)$ can be calculated by the standard methods. For $y \to \pm\infty$ it has the asymptotic behavior

$$Q(y) \to \frac{C}{y^2} \tag{C3}$$

and its second moment diverges. The general theory of broad distributions [21–23] predicts that in the limit $N \to \infty$ the probability density of the sum (C1) $Y = 1/X$ is Lorenzian

$$Q_N(Y) = \frac{1}{\pi}\frac{\pi C}{(\pi C)^2 + Y^2} \tag{C4}$$

and consequently the probability density $P_N(X))$ of variable $X = 1/Y$ is again a Lorenzian:

$$P_N(X) = \frac{1}{\pi}\frac{\delta}{\delta^2 + X^2} \tag{C5}$$

characterized by the parameter δ related to the original distribution $P(x)$ by the formula:

$$\delta = \frac{1}{\pi C} \tag{C6}$$

In particular for the uniform distribution (33) it takes the value $\delta = \Delta/\pi$.

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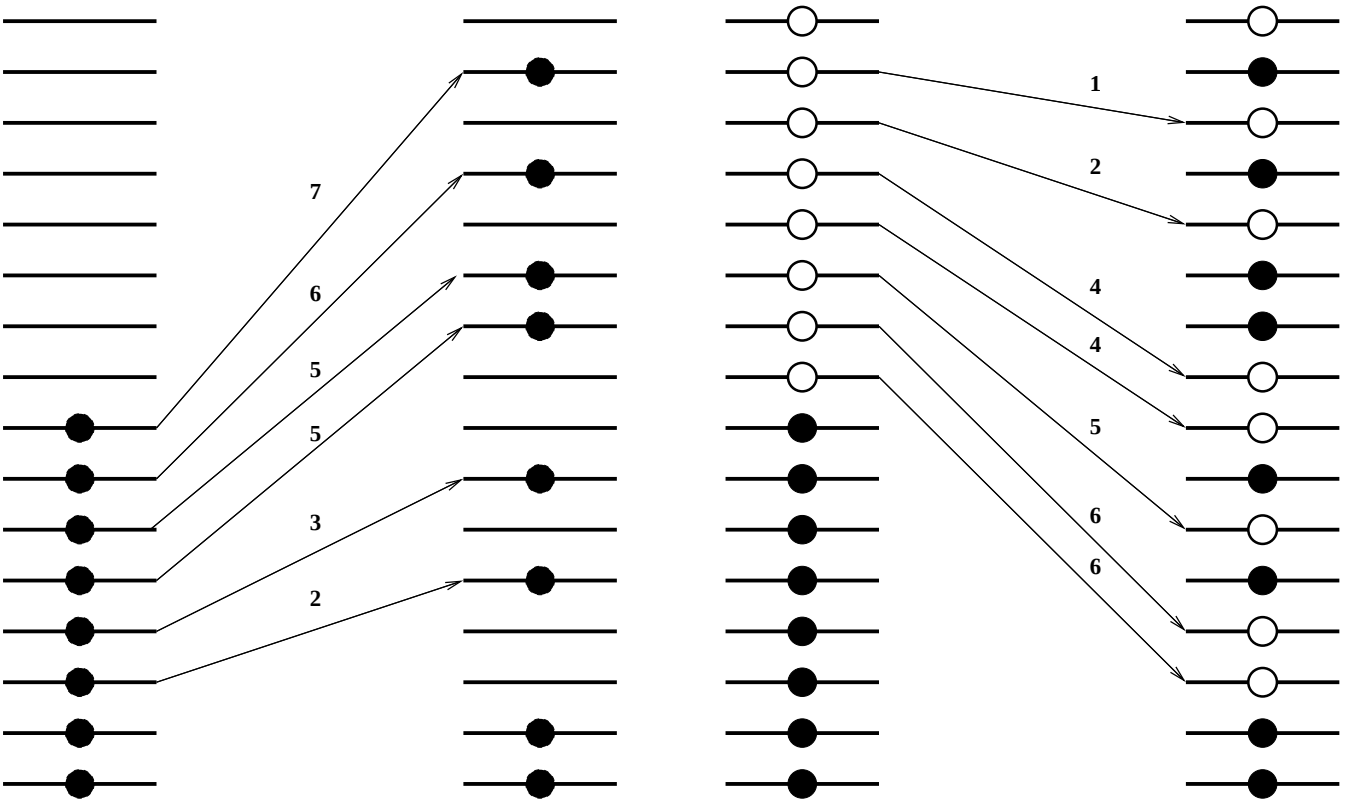

FIG. 1. Correspondence between Slater determinant and partition $(\lambda) = (7, 6, 5, 5, 3, 2)$. The dual partition $(\tilde{\lambda}) = (6, 6, 5, 4, 4, 2, 1)$ represents shifts of the holes.

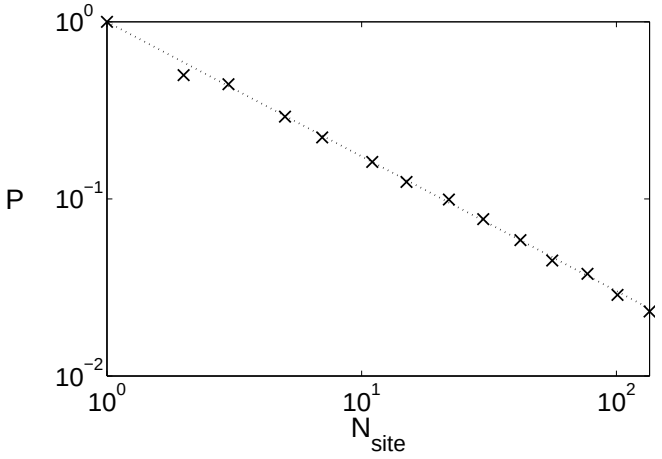

FIG. 2. The inverse participation ratio (19) as function of number of sites $N_{site} = p(N)$. The solid line represents the fit $P = 1/N_{site}^{\alpha}$, where $\alpha \approx 0.8$

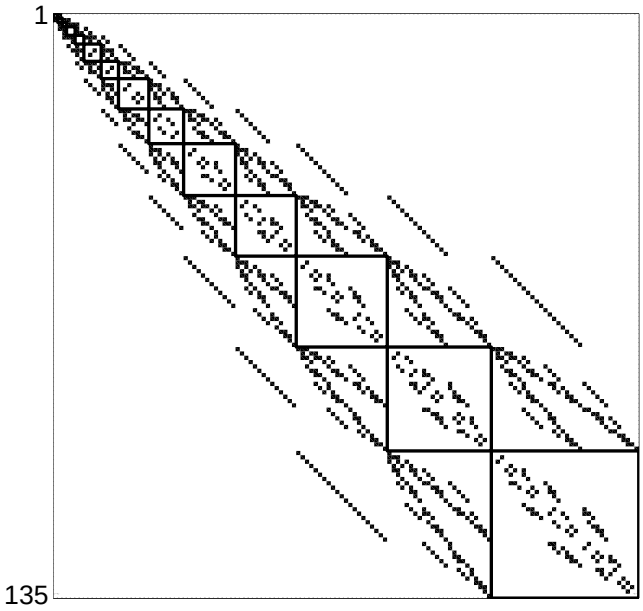

FIG. 3. The matrix of the interaction (23). The blocks corresponding to different groups of partitions are shown.

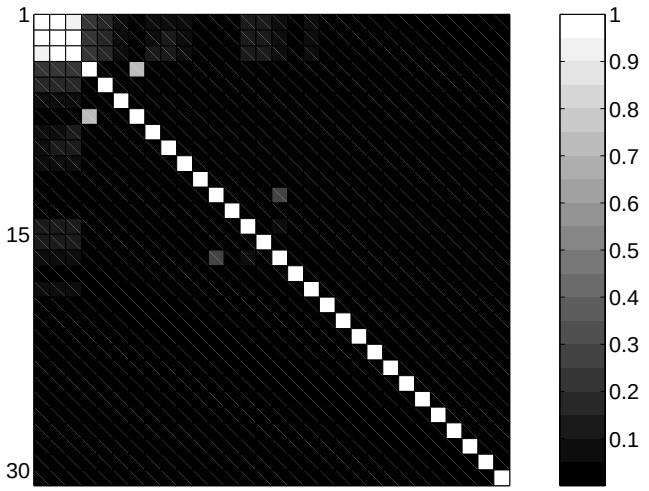

FIG. 4. The absolute value of correlation matrix elements C_{kl} defined in (43) of effective energies $\mathcal{E}_k$ for $N = 30$.

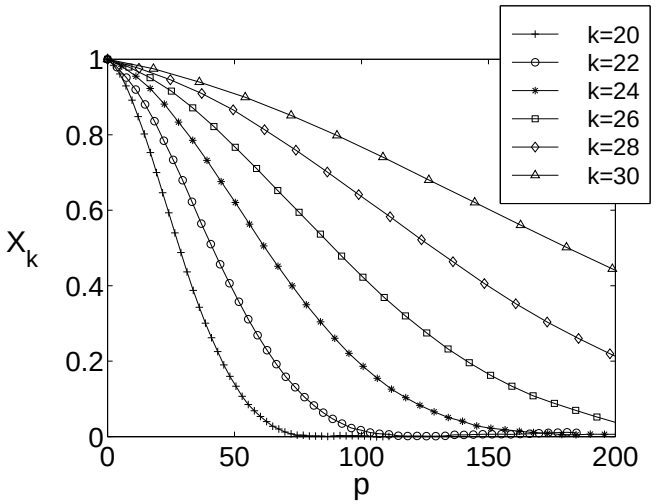

FIG. 5. Characteristic function $X_k(p)$ of the effective energy distribution for some values of k.

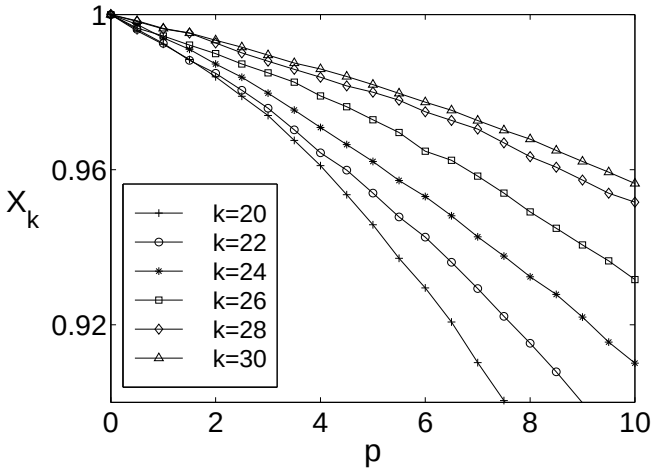

FIG. 6. The same as on the previous figure for a smaller range of p.

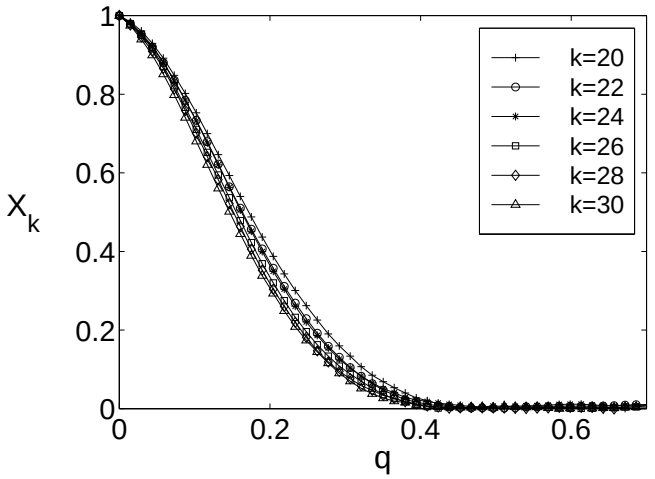

FIG. 7. The characteristic function $X_k(p)$ as a function of the scaled variable $q = p\Delta/n(k)$ for values of k used in Figs 5-6